

Problem dveh teles

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2. Newtonov zakon za dve (sicer) prosti telesi

- $m_1 \ddot{\vec{r}}_1 = \vec{F}_{2na1}$
- $m_2 \ddot{\vec{r}}_2 = \vec{F}_{1na2}$

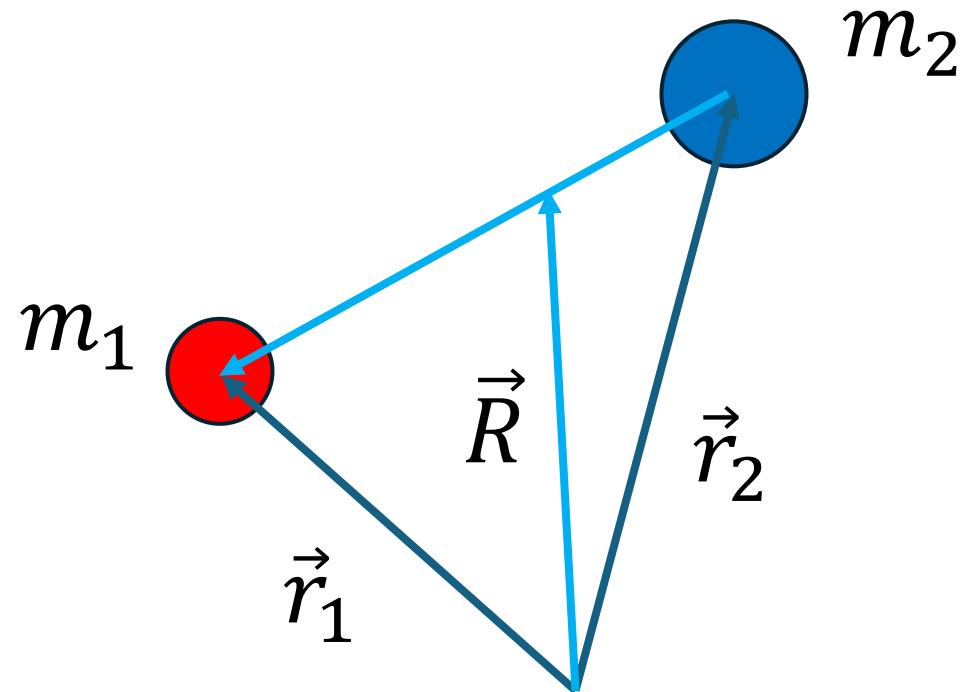
- 3. Newtonov zakon: $\vec{F}_{1na2} = -\vec{F}_{2na1}$

- $m_1 \ddot{\vec{r}}_1 + m_2 \ddot{\vec{r}}_2 = (m_1 + m_2) \ddot{\vec{R}} = 0$

- $\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$

- hitrost težišča je konstanta – ohranitev gibalne količine

- problem enega telesa



2. Newtonov zakon za dve (sicer) prosti telesi

- $\ddot{\vec{r}}_1 - \ddot{\vec{r}}_2 = \ddot{\vec{r}} = \frac{1}{\mu} \vec{F}(\vec{r}_1, \vec{r}_2) = \frac{1}{\mu} \vec{F}(\vec{r})$ – translacijska simetrija

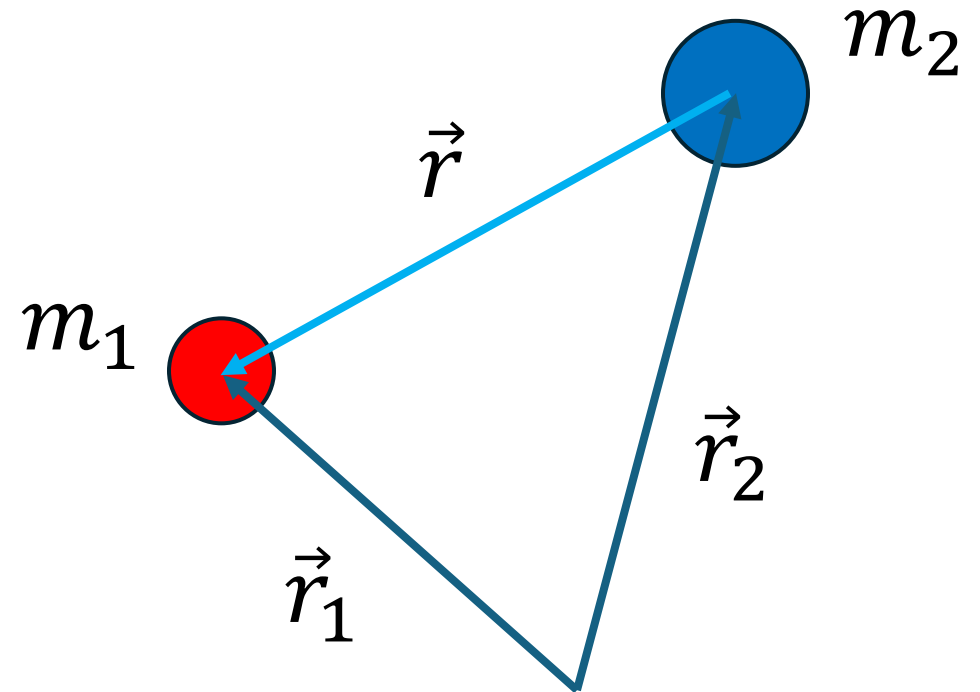
- $\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$

- $\ddot{\vec{r}} = \frac{1}{\mu} \vec{F}(\vec{r})$

- problem enega telesa

- $\vec{r}_1 = \vec{R} + \frac{m_2}{m_1 + m_2} \vec{r}$

- $\vec{r}_2 = \vec{R} - \frac{m_1}{m_1 + m_2} \vec{r}$



gibanje je ravninsko

- $\vec{r} \times \dot{\vec{r}} = \vec{H}$, H je specifična vrtilna količina, pravokotna na ravnino, ki jo tvorita vektor r in vektor hitrosti
- časovni odvod te količine $\dot{\vec{r}} \times \dot{\vec{r}} + \vec{r} \times \ddot{\vec{r}} = 0 + 0$ (centralna sila)
- vrtilna količina se ne spreminja, kar pomeni, da vektor r in hitrost stalno ležita v isti ravnini.

Keplerjev problem/orbite

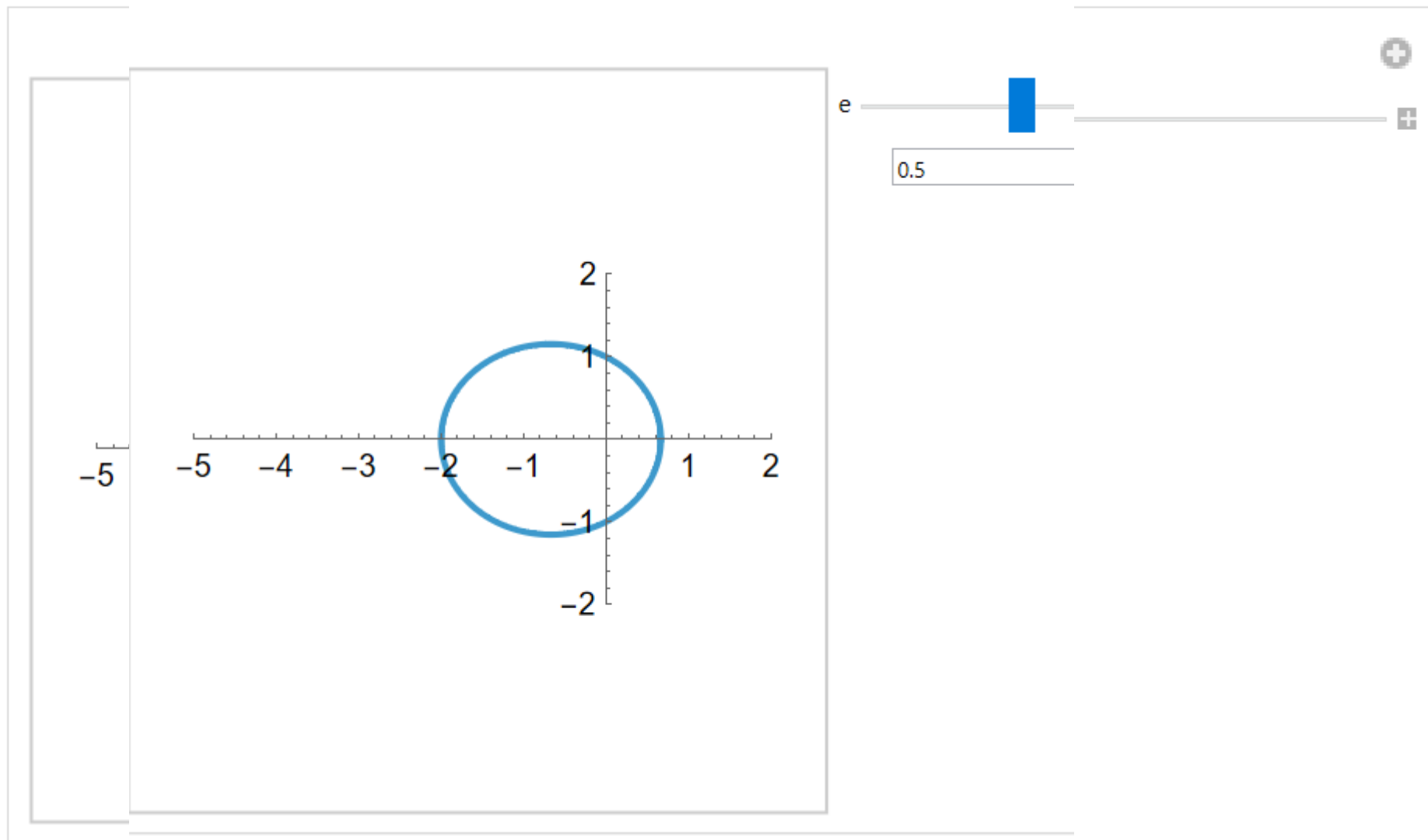
- $\vec{F} = \frac{k}{r^2} \vec{e}_r, \vec{r} = r \vec{e}_r$
- $\ddot{\vec{r}} = -\frac{\alpha}{r^2} \vec{e}_r, \alpha = G(m_1 + m_2)$
- $\vec{H} = r^2 \vec{e}_r \times \dot{\vec{e}}_r$
- $\ddot{\vec{r}} \times \vec{H} = -\alpha [(\vec{e}_r \cdot \dot{\vec{e}}_r) \vec{e}_r - (\vec{e}_r \cdot \dot{\vec{e}}_r) \dot{\vec{e}}_r]$
- $\text{ker } \vec{e}_r \cdot \vec{e}_r = 1, \vec{e}_r \cdot \dot{\vec{e}}_r = \frac{1}{2} \frac{d}{dt} (\vec{e}_r \cdot \vec{e}_r) = 0$
- $\ddot{\vec{r}} \times \vec{H} = \alpha \dot{\vec{e}}_r$

Keplerjev problem/orbite

- $\dot{\vec{r}} \times \vec{H} = \alpha \vec{e}_r + \vec{c}$
- $\vec{r} \cdot (\dot{\vec{r}} \times \vec{H}) = \alpha r \vec{e}_r \cdot \vec{e}_r + r c \cos \theta$
- $r = \frac{\vec{r} \cdot (\dot{\vec{r}} \times \vec{H})}{\alpha + c \cos \theta} = \frac{\vec{r} \times \dot{\vec{r}} \cdot \vec{H}}{\alpha + c \cos \theta} = \frac{H^2}{\alpha + c \cos \theta} = \frac{p}{1 + e \cos \theta}$

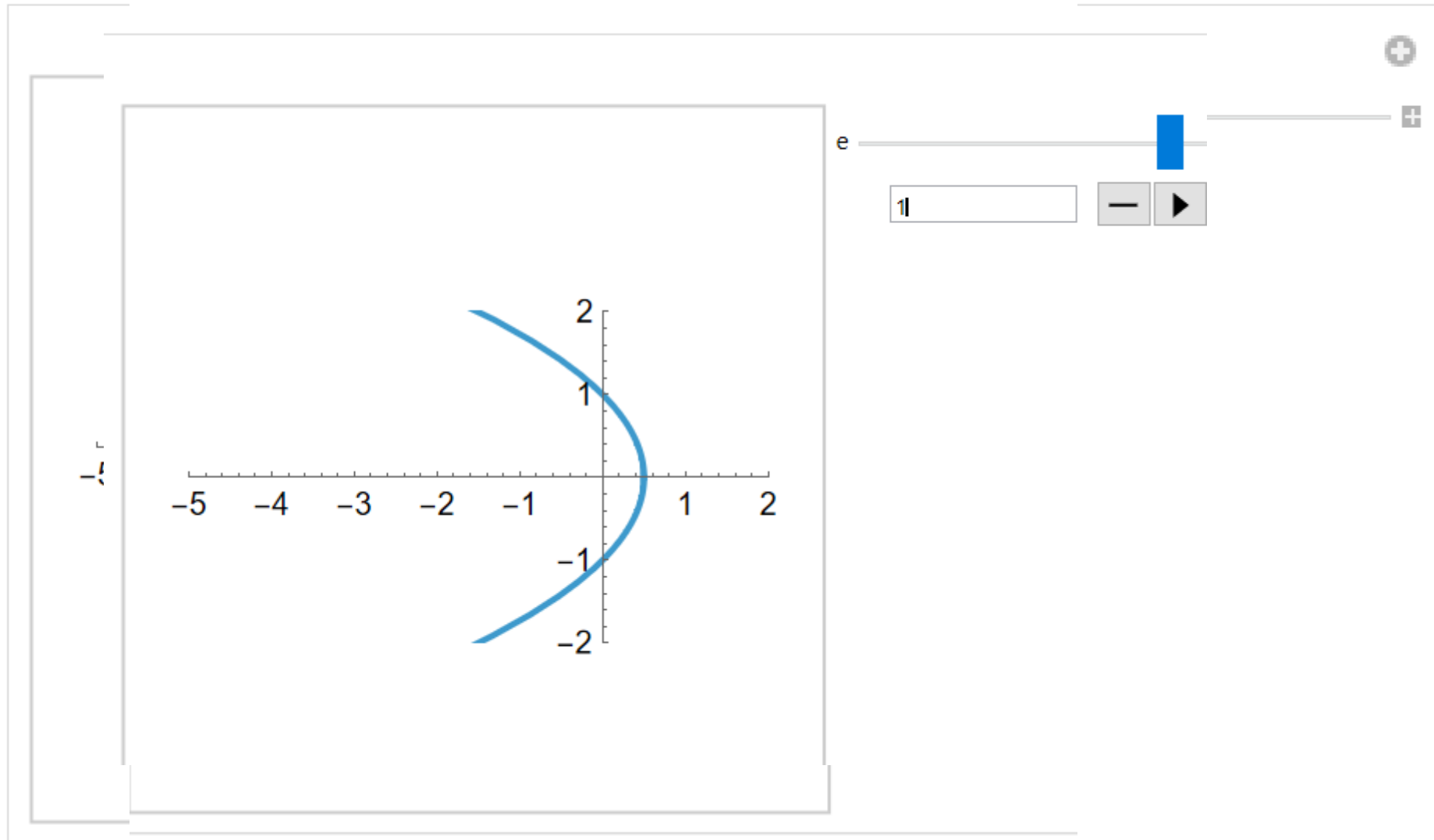
```
In[10]:= Manipulate[PolarPlot[1 / (1 + e Cos[t]), {t, 0, 2 Pi},  
PlotRange -> {{-5, 2}, {-2, 2}}], {e, 0, 2}]
```

Out[10]=

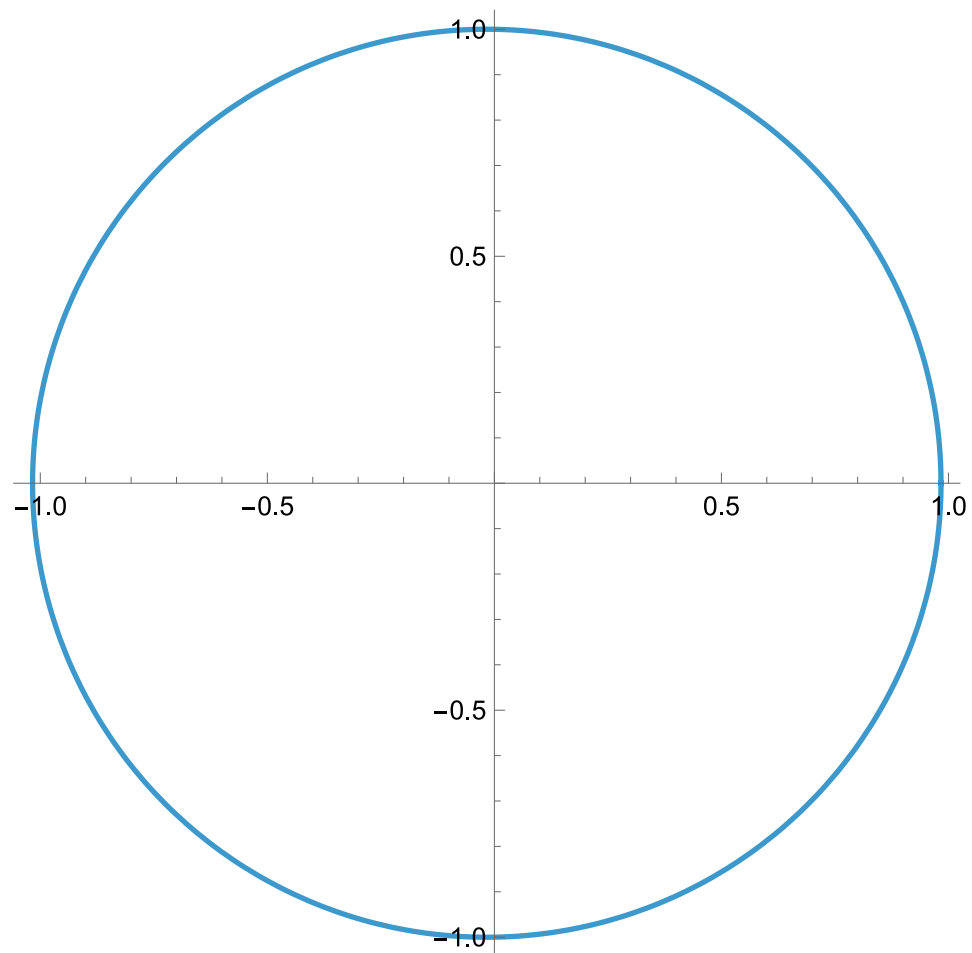


```
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PlotRange -> {{-5, 2}, {-2, 2}}], {e, 0, 2}]
```

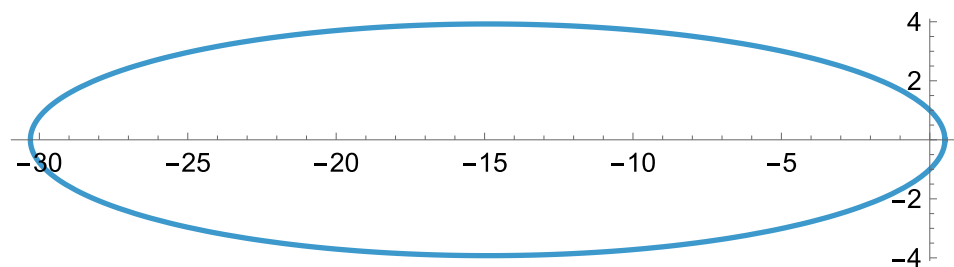
Out[10]=



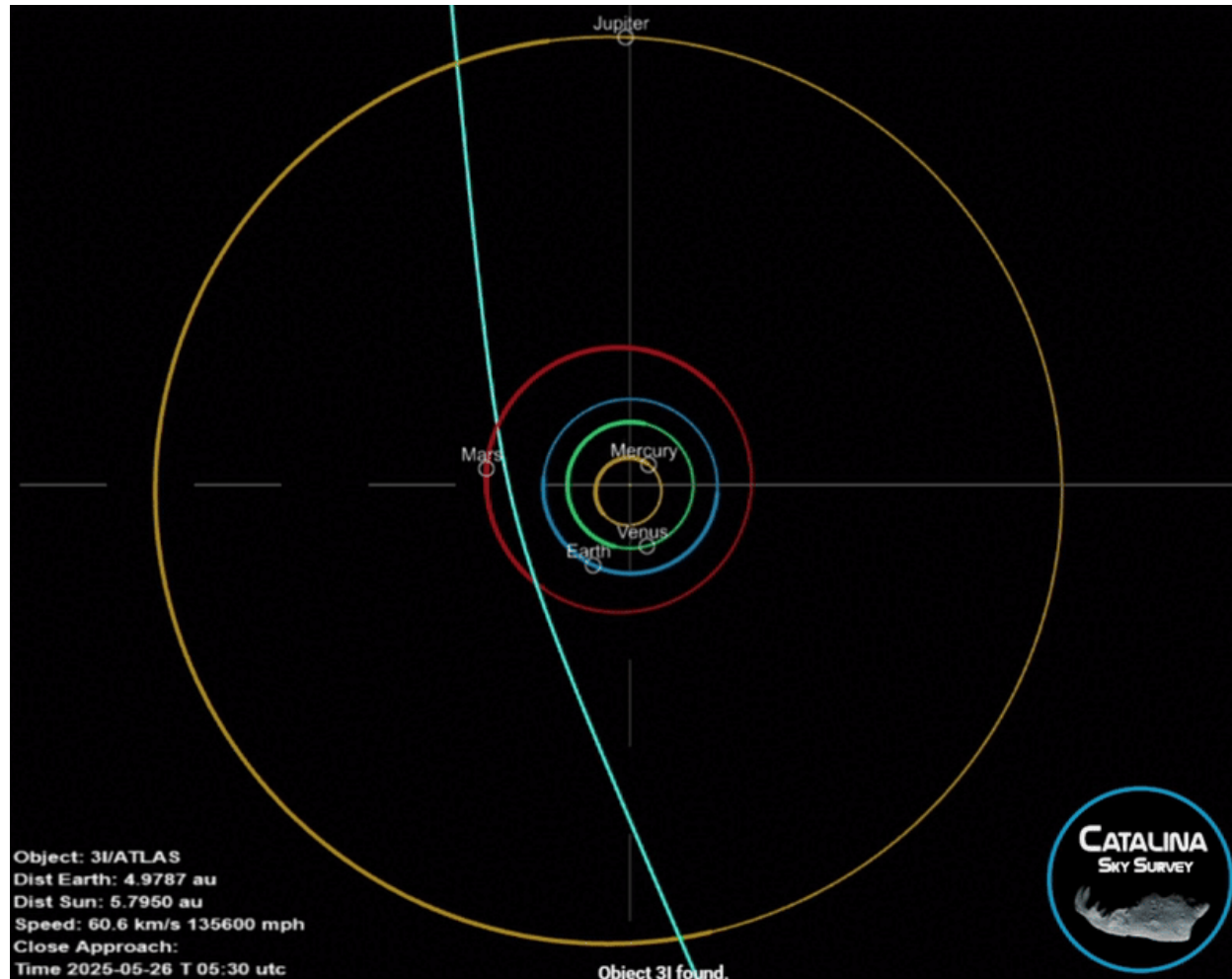
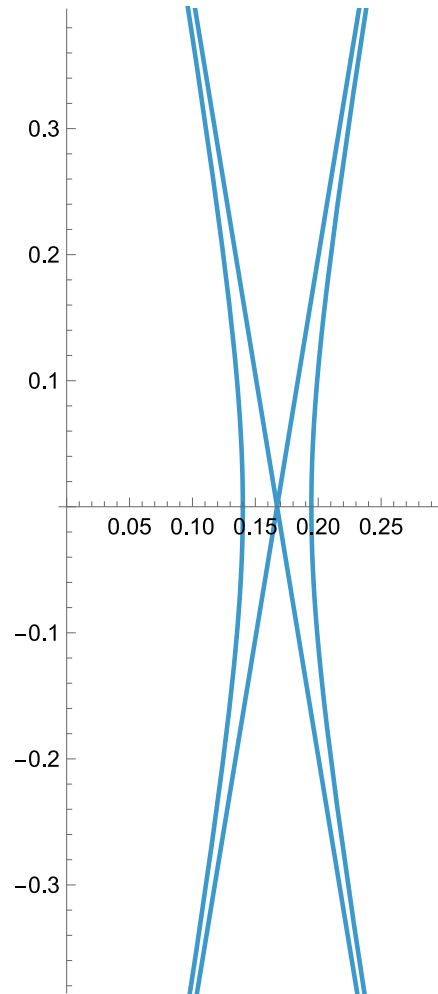
$e=0,0167$ - Zemlja



$e = 0,967$ – Halleyev komet



$e = 6,14$ - medzvezdni komet 3I/ATLAS



Copilot prompt: mathematica expression for animation of kepler orbits of two stars. include plot of orbits

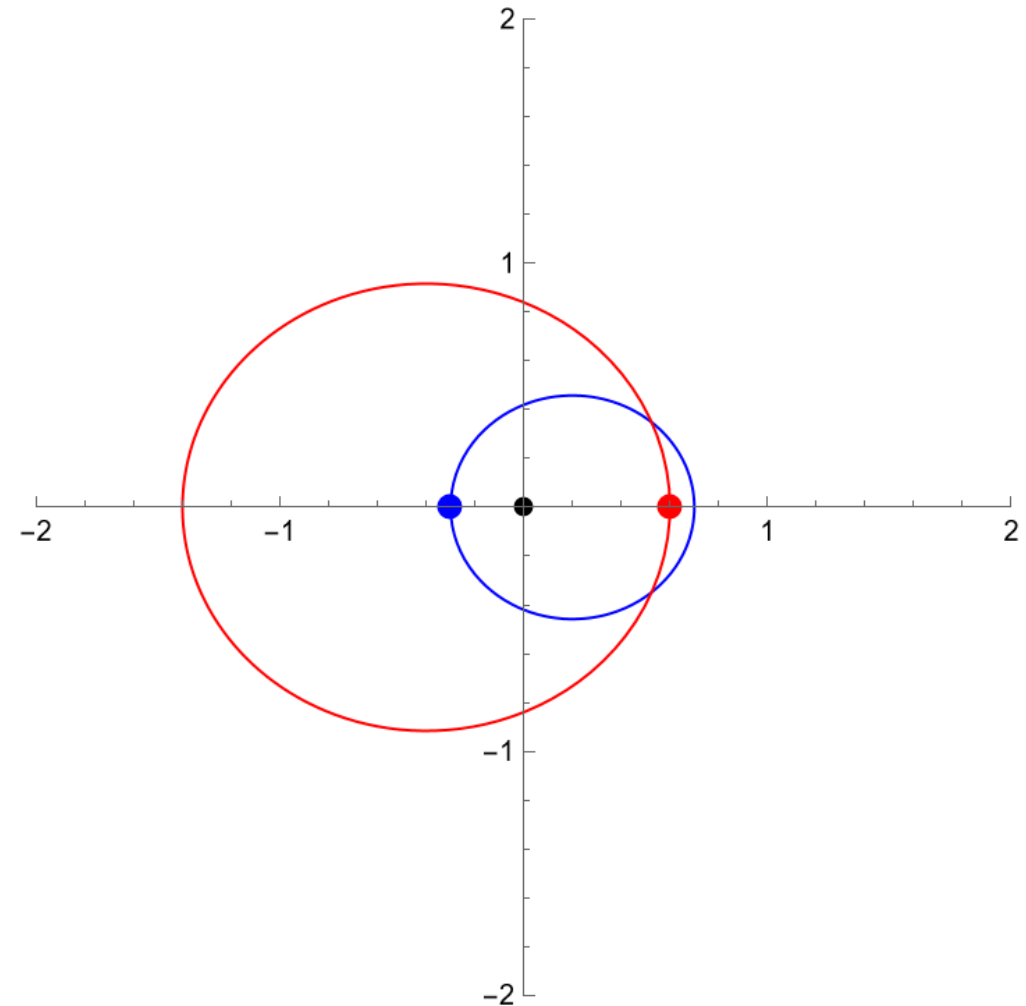
```
(* Parameters *)
m1 = 2; m2 = 1; (* masses of the two stars *)
a = 1.5; (* semi-major axis of relative orbit *)
e = 0.4; (* eccentricity *)

(* Function to solve Kepler's equation and get position *)
keplerOrbit[a_, e_, M_] := Module[{E},
  E /. FindRoot[E - e Sin[E] == M, {E, M}] //
  ({a (Cos[#] - e), a Sqrt[1 - e^2] Sin[#]} &)
];

(* Positions of each star relative to barycenter *)
starPos[m1_, m2_, a_, e_, M_] := Module[{r},
  r = keplerOrbit[a, e, M];
  {-(m2/(m1 + m2)) r, (m1/(m1 + m2)) r}
];

Animate[
Module[{pos1, pos2, orbit1, orbit2},
{pos1, pos2} = starPos[m1, m2, a, e, t];
orbit1 = Table[starPos[m1, m2, a, e, M][[1]], {M, 0, 2 Pi, 0.05}];
orbit2 = Table[starPos[m1, m2, a, e, M][[2]], {M, 0, 2 Pi, 0.05}];

Graphics[
{
Black, PointSize[Large], Point[{0, 0}], (* barycenter *)
Blue, Disk[pos1, 0.05], Line[orbit1], (* star 1 and its orbit *)
Red, Disk[pos2, 0.05], Line[orbit2] (* star 2 and its orbit *)
},
PlotRange -> {{-2, 2}, {-2, 2}},
Axes -> True
]
,
{t, 0, 2 Pi}
]
```



- https://en.wikipedia.org/wiki/Two-body_problem
- https://en.wikipedia.org/wiki/Kepler_problem